SOLUCIONARIO DEL EXAMEN PARCIAL DE BASICAS II

Ciclo: 2017-I
UNI-FIA

1) Dato: $\mathcal{A} = \{ M \in \mathbb{R}^{n\times n} \,/\, traz(M) = n \}$

p: $\mathcal{A}$ es un conjunto nulo (1pt)
En efecto
Si $M = I_{n\times n} \longrightarrow traz(I) = n$ (V)

q: $A, B \in \mathcal{A}$ entonces $A+B \in \mathcal{A}$
En efecto.
$traz(A+B) = traz(A) + traz(B) = n + n = 2n \longrightarrow A+B \notin \mathcal{A}$ (F) (1pt)

r: $\exists A, B \in \mathcal{A}$ tq. $A.B \in \mathcal{A}$.
En efecto
tomando: $B = I \in \mathcal{A}$
$A \in \mathcal{A}$ (1pt)
$traz(I.A) = traz(A) = n \rightarrow I.A \in \mathcal{A}$. (V).

2.- Datos: $A_{n\times n}$; $|A| \neq 0$
$f(x) = |A - xI| \;\wedge\; g(x) = |A^{-1} - xI|$ ---(1)
$g(x) = h(x) \cdot f(\frac{1}{x})$ --(2)

de (1) $\wedge$ (2) reemplazando

$\underline{|A^{-1} - xI|} = h(x)\, |A - \frac{1}{x} I|$

$|A^{-1}xx^{-1} - xA^{-1}A| = h(x)\, |A - \frac{1}{x} I|$

$|-A^{-1}x\,(-x^{-1}I + A)| = h(x)\, |A - \frac{1}{x} I|$

$|-A^{-1}x|\; \cancel{|A - x^{-1}I|} = h(x)\; \cancel{|A - \frac{1}{x} I|} \longrightarrow |(-x)\, A^{-1}| = h(x)$

$h(x) = (-x)^n\, |A^{-1}| \rightarrow h(x) = \dfrac{(-1)^n x^n}{|A|}$ (4pt)

3.- Datos
* A es triangular superior
* B es simétrica $B = B^t$ ---(1)
* $AB = BA$ --(2)
* $A^t.B.A = \begin{bmatrix} 1/4 & 3/2 \\ 3/2 & 13 \end{bmatrix} . B^t$ ...(3)

Solución

En (3). $A^t.(A.B) = \begin{bmatrix} 1/4 & 3/2 \\ 3/2 & 13 \end{bmatrix} B^t$ m.a.m B^{-1} (por la derecha)

$A^t(A.\underbrace{BB^{-1}}_{I}) = \begin{bmatrix} 1/4 & 3/2 \\ 3/2 & 13 \end{bmatrix} \underbrace{BB^{-1}}_{I}$

$A^t \underbrace{AI}_{A} = \begin{bmatrix} 1/4 & 3/2 \\ 3/2 & 13 \end{bmatrix} \longrightarrow A^tA = \begin{bmatrix} 1/4 & 3/2 \\ 3/2 & 13 \end{bmatrix}$ ...(α)

///.

//// haciendo: $A=\begin{bmatrix} a & b \\ 0 & c\end{bmatrix}$; $a,b,c \in \mathbb{R}^+ \to A^t=\begin{bmatrix} a & 0\\ b & c\end{bmatrix}$

reempl. en (α)

$$\begin{bmatrix} a&0\\b&c\end{bmatrix}\begin{bmatrix}a&b\\0&c\end{bmatrix} = \begin{bmatrix}1/4 & 3/2\\ 3/2 & 13\end{bmatrix} \to \begin{bmatrix}a^2 & ab\\ ab & b^2+c^2\end{bmatrix}=\begin{bmatrix}1/4&3/2\\3/2&13\end{bmatrix}$$

$a^2=1/4 \to a=1/2$

$ab=3/2 \to b=3$

$b^2+c^2=13 \to c=2$

$A=\begin{bmatrix}1/2&3\\0&2\end{bmatrix}$ (4pts)

4) Dato $|A^{-1}|>0 \to \frac{1}{|A|}>0 \to |A|>0$

$$Adj(A^{-1}) = \begin{bmatrix}2&3&1\\1&-1&3\\1&0&1\end{bmatrix} \ldots (1)$$

En (1) aplicando determinante m.a.m.

$$\underbrace{|Adj(A^{-1})|} = \begin{vmatrix}2&3&1\\1&-1&3\\1&0&1\end{vmatrix}$$

$||A^{-1}|.A| = 5 \to |A^{-1}|^3|A|=5 \to |A|^{-3+1}=5 \to |A|^{-2}=5 \to \frac{1}{|A|^2}=5$

$|A| = \pm\frac{1}{\sqrt5} \to |A|=\frac{1}{\sqrt5}$

de (1) $|A^{-1}|A = \begin{bmatrix}2&3&1\\1&-1&3\\1&0&1\end{bmatrix} \to A = |A|\begin{bmatrix}2&3&1\\1&-1&3\\1&0&1\end{bmatrix} \to A = \begin{bmatrix}\frac{2}{\sqrt5}&\frac{3}{\sqrt5}&\frac{1}{\sqrt5}\\ \frac{1}{\sqrt5}&-\frac{1}{\sqrt5}&\frac{3}{\sqrt5}\\ \frac{1}{\sqrt5}&0&\frac{1}{\sqrt5}\end{bmatrix}$

luego reempl.

$$\begin{bmatrix}\frac{2}{\sqrt5}&\frac{3}{\sqrt5}&\frac{1}{\sqrt5}\\ \frac{1}{\sqrt5}&-\frac{1}{\sqrt5}&\frac{3}{\sqrt5}\\ \frac{1}{\sqrt5}&0&\frac{1}{\sqrt5}\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix} = \begin{bmatrix}4\\2\\1\end{bmatrix} \to \begin{cases}2x+3y+z=4\sqrt5\\ x-y+3z=2\sqrt5\\ x+z=\sqrt5\end{cases}$$

Resolviendo

$x=0$; $y=\sqrt5$

$z=\sqrt5$

$C.S=\{(0;\sqrt5,\sqrt5)\}$

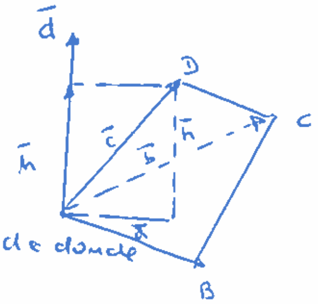
(4 pts)

5) a) Sean $\bar a=\overline{AB}=(2;-2;-3)$

$\bar b=\overline{AC}=(4;0;6)$

$\bar c=\overline{AD}=(-7;-7;7)$

$\bar d=\bar a\times\bar b=(-12;-24;8)$



Sea $\|\bar h\|$ la altura basada. Luego $\bar h = proy_{\bar d}\bar c$ de donde

$$\|\bar h\| = \|proy_{a}\bar c\| = |comp_{\bar d}\bar c| = \frac{|\bar c.\bar d|}{\|\bar d\|} = \frac{302}{\sqrt{784}} = \frac{308}{28} = 11 \to \|\bar h\|=11$$

(3pts)

b) $(\bar B+\bar C)\times(\bar C+\bar A) = \bar B\times(\bar C+\bar A)+\bar C\times(\bar C+\bar A) = \bar B\times\bar C+\bar B\times\bar A+\bar C\times\bar C+\bar C\times\bar A$

$= \bar B\times\bar C+\bar B\times A+\bar C\times\bar A = (\bar A+\bar B).(\bar B+\bar C)\times(\bar C+\bar A) = (\bar A+\bar B).(\bar B\times\bar C+\bar B\times\bar A+\bar C\times\bar A)$

$= \bar A.(\bar B\times\bar C+\bar B\times\bar A+\bar C\times\bar A)+\bar B.(\bar B\times\bar C+\bar B\times\bar A+\bar C\times\bar A)$

$= \bar A.\bar B\times\bar C+\bar A.\bar B\times\bar A.\bar A.\bar C\times\bar A+\bar B.\bar B\times\bar C+\bar B.\bar B\times\bar A+\bar B.\bar C\times\bar A$

$= \bar A.\bar B\times\bar C+\bar 0+\bar 0+\bar 0+\bar 0+\bar B.\bar C\times\bar A = \bar A.\bar B\times\bar C+\bar C.\bar A\times\bar B$

(2pts)

$= \bar A.\bar B\times\bar C+\bar A.\bar B\times\bar C = 2\bar A.\bar B\times\bar C$